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# Flatten and Conquer

A Framework for Efficient Analysis of  
String Constraints

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<sup>1</sup> Uppsala University, Sweden

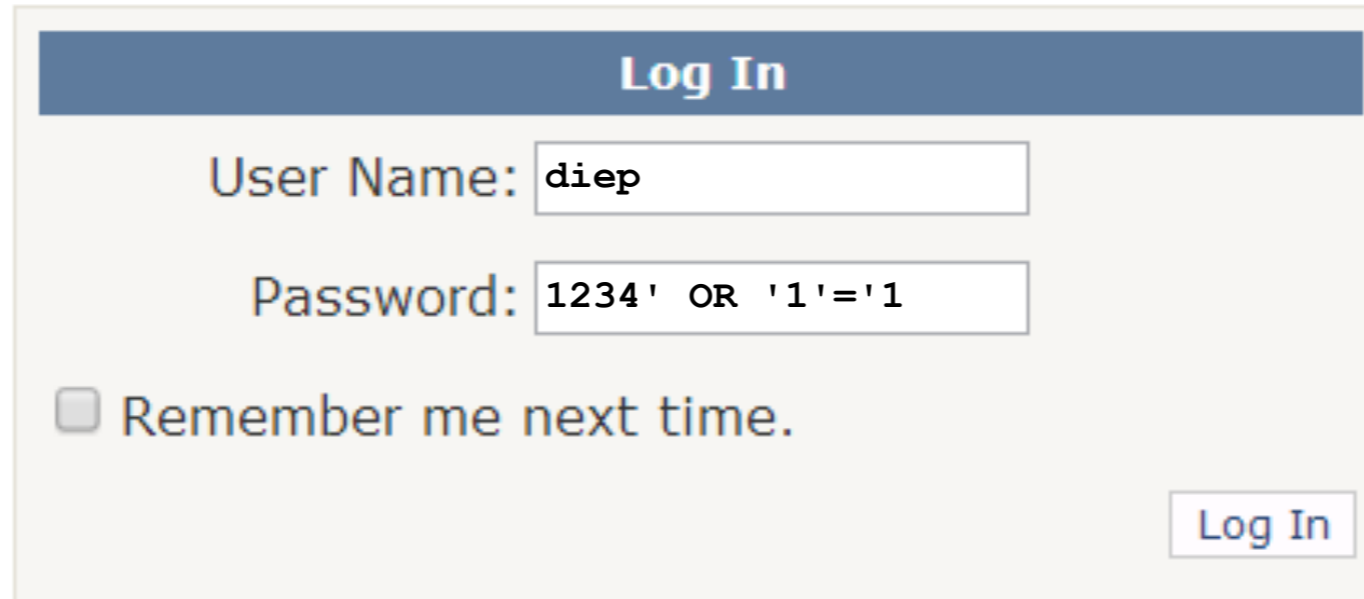
<sup>2</sup> Academia Sinica, Taiwan

<sup>3</sup> Brno University of Technology, Czech Republic

<sup>4</sup> Linköping University, Sweden

# SQL Injections

- ❑ Consider a webpage that has two input fields: **username** and **password**



Log In

User Name:

Password:

Remember me next time.

ACCESS GRANTED

- ❑ The code behind the webpage is the following:

```
void Login_Authenticate(object sender, AuthenticateEventArgs e){
    SqlConnection con = new SqlConnection(@"Data Source=. \sqlexpress;Initial Catalog=...=True");
    string stmt = "select * from Table where name = '" + Name + "' and passwd = '" + Pas...";
    adpt = new SqlDataAdapter(qry, con);
    dt = new DataTable();
    adpt.Fill(dt);
    if (dt.Rows.Count >= 1){
        select * from Table where name = 'diep' and passwd = '1234' OR '1'='1'
```

always TRUE

# Detect SQL Injection via String Constraints

## Step 1: Identify variables

```
stmt = "select * from Table where name = '\" + Name + \"' and passwd = '\" + Passwd + \"'\"";
```

## Step 2: Find forbidden patterns

```
SQL_QUERY : "select * from Table where " EXPR  
EXPR : COMP | EXPR or EXPR  
COMP : TERM (=|>|<) TERM  
TERM : [a-z]+[0-9]* | [0-9]+ | 'TERM'
```

SQL injection is to create a SQL query that contains " or '1' = '1' "

context free  
grammar

## Step 3: Transform to string constraints

## Step 4: Solve the string constraints

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SQL injection is to create a **SQL query** that contains " or '1' = '1' "

## Step 3: Transform to string constraints

```
stmt = "select * from Table where name = '\" . Name .  
\"' and passwd = '\" . Passwd . '\""
```

```
SQL_injection ∈ L(SQL_QUERY);
```

```
SQL_injection = A . " or '1' = '1' " . B
```

```
stmt = SQL_injection
```

concatenation

membership

equality

## Step 4: Solve the string constraints

# String Solver

## ✓ Applications

- Detect **vulnerabilities** in web applications
  - SQL Injection
  - Code Injection
- Used in **Program Testing, Program Verification, Model Checking**

## ✓ Requirements

- Arithmetic constraints `length(A) > 5`
- String equations `stmt = A . " or '1 = 1'" . B`
- Context free grammar membership `stmt ∈ L(SQL_QUERY);`

...

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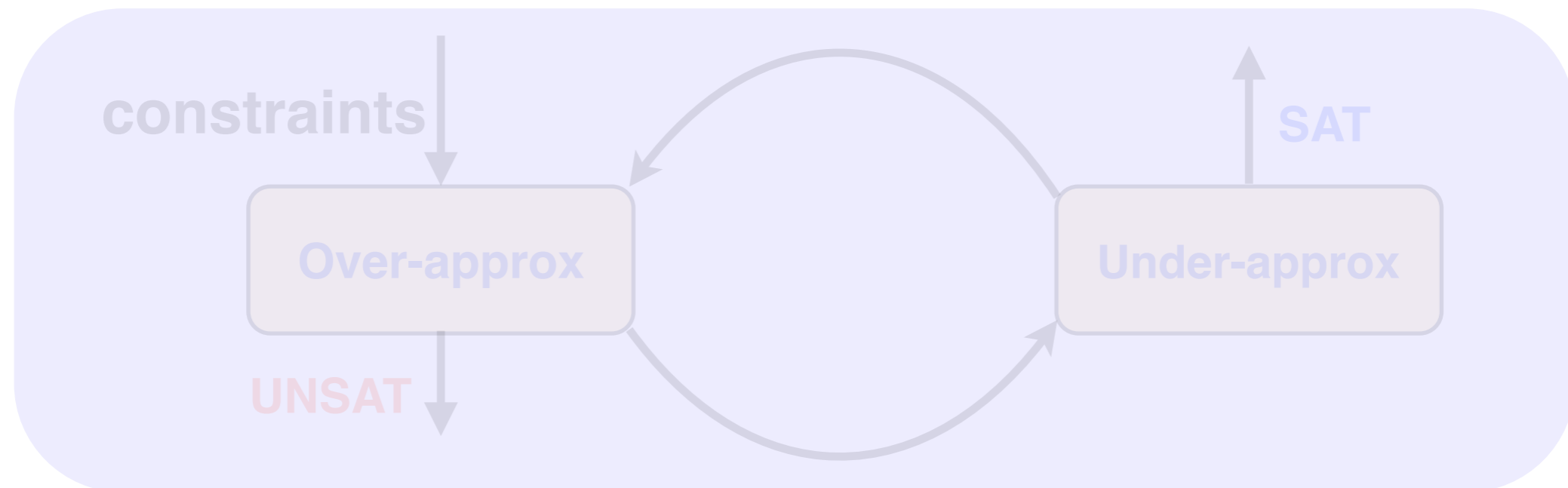
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# Contributions

## 1. New framework for solving **string constraints**:

- Handle **rich** class of constraints: CFG membership, transducer, etc.
- Based on Counter-Example Guided Abstract Refinement.



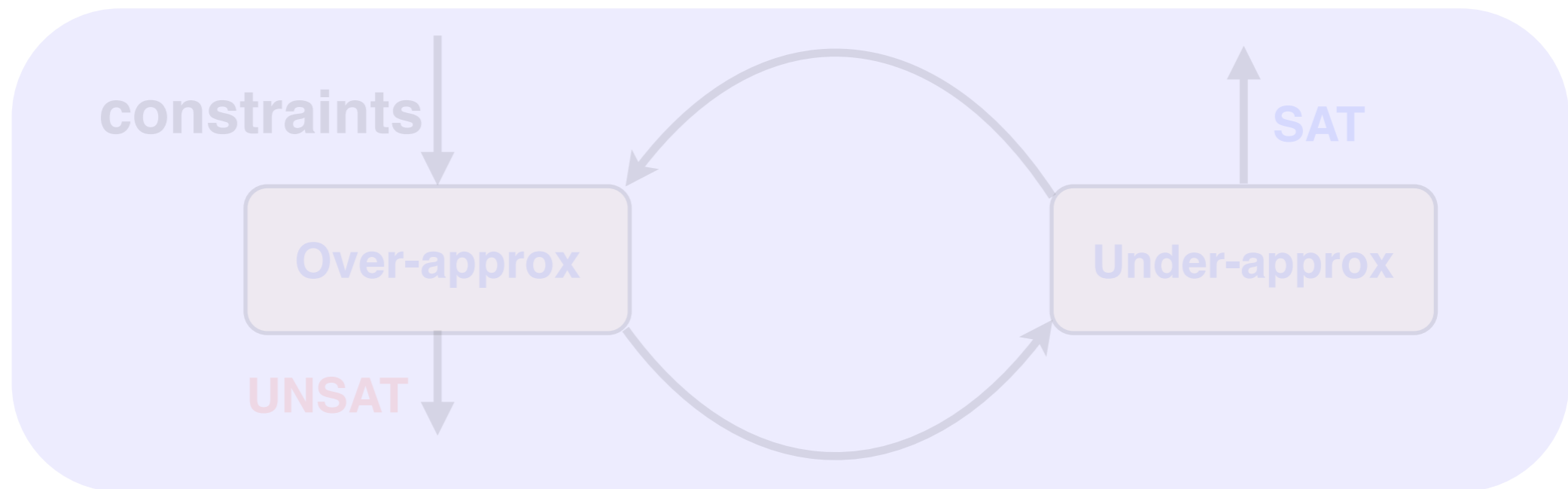
## 2. Open-source tool: **outperforming** all existing tools.

no other tools  
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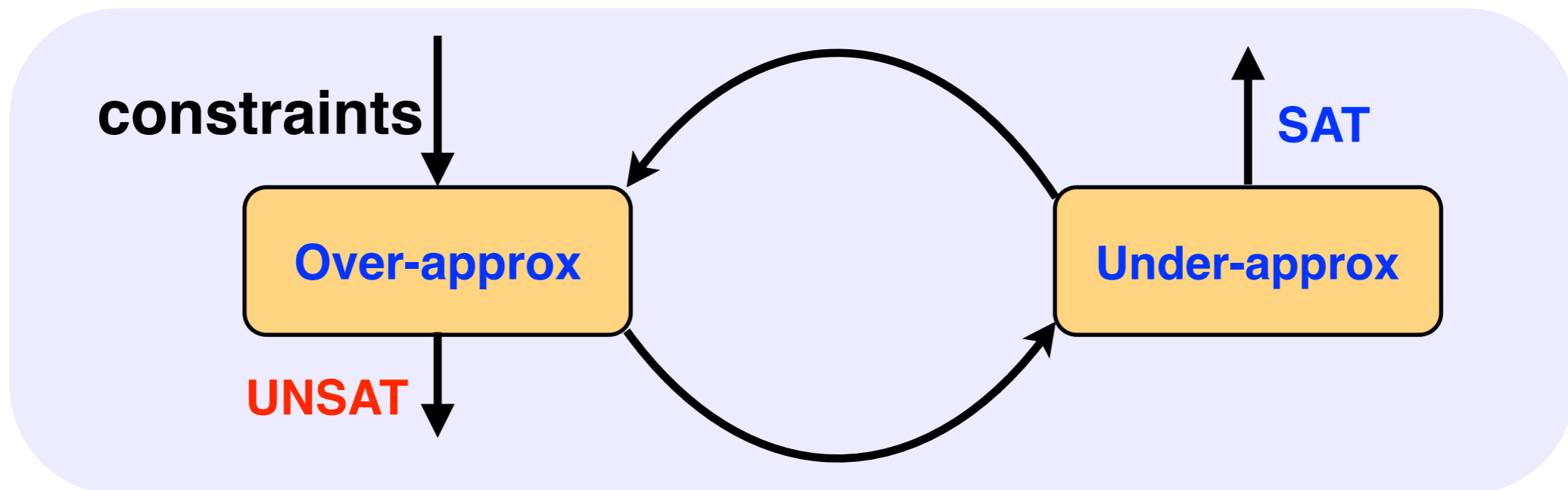


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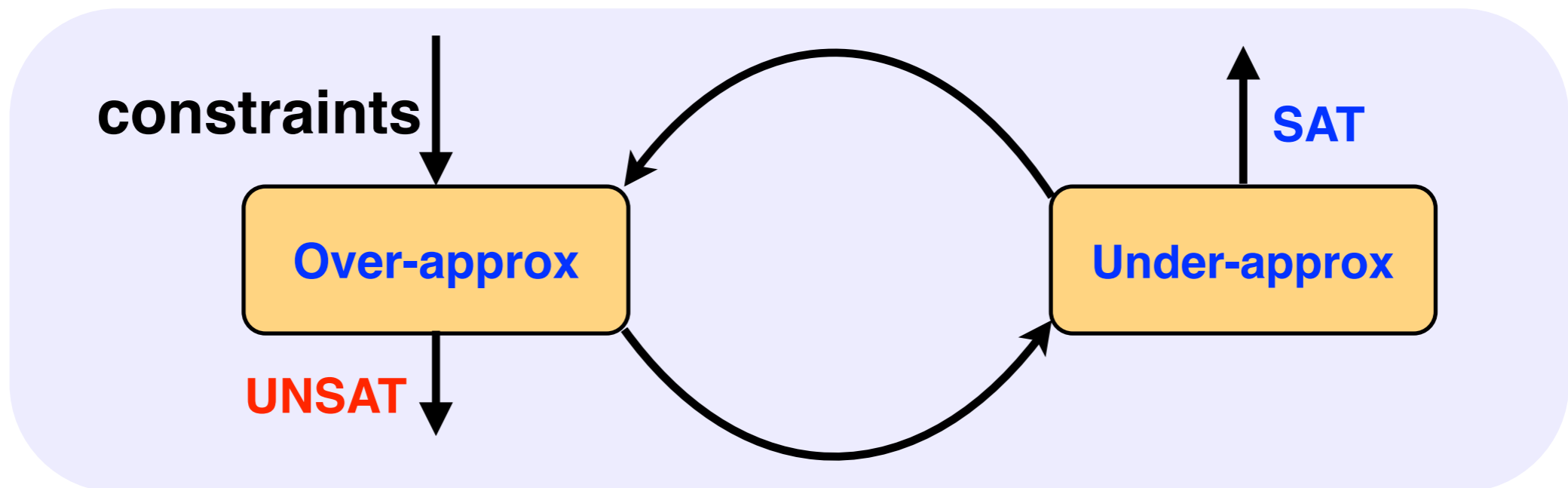
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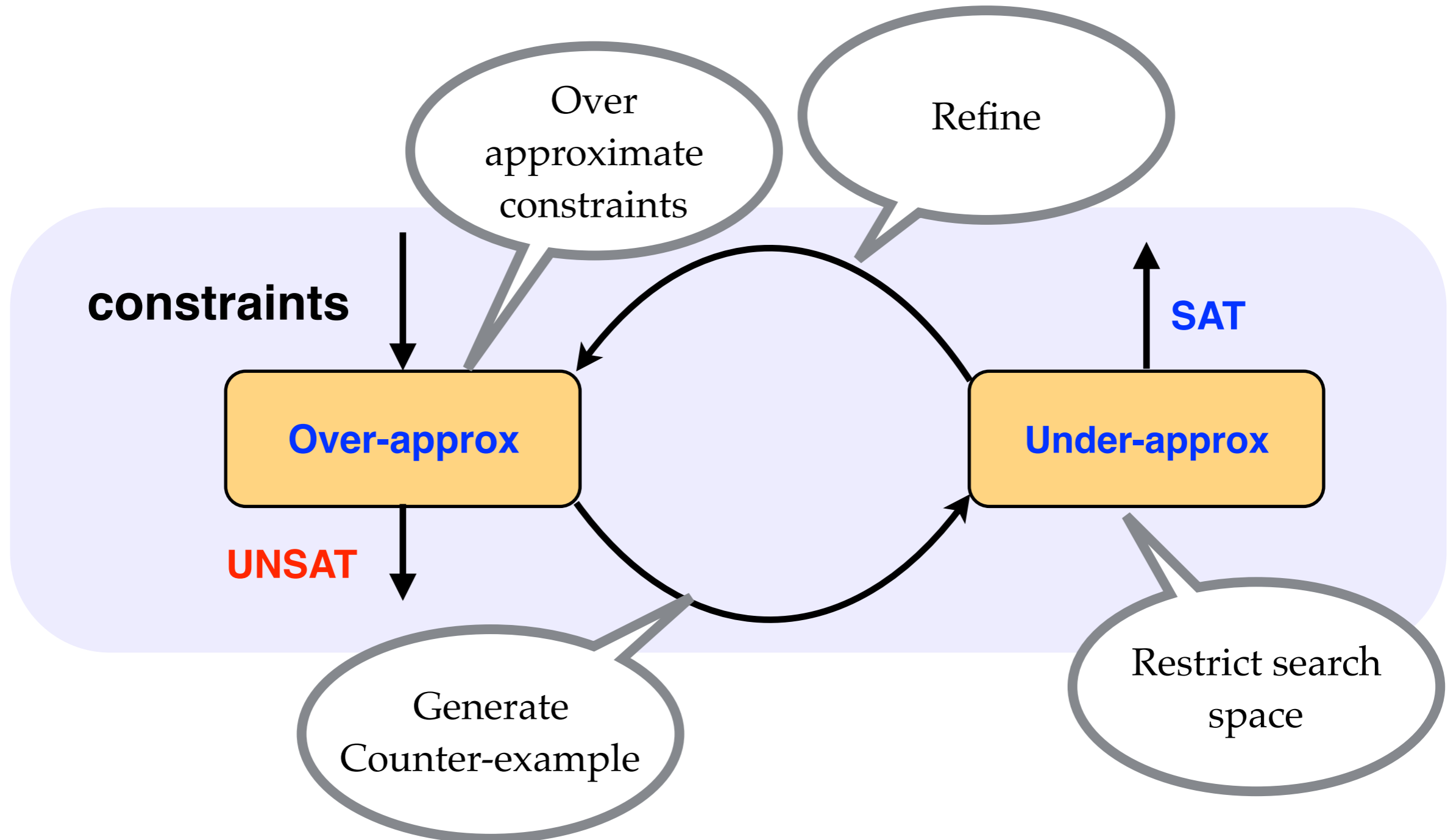
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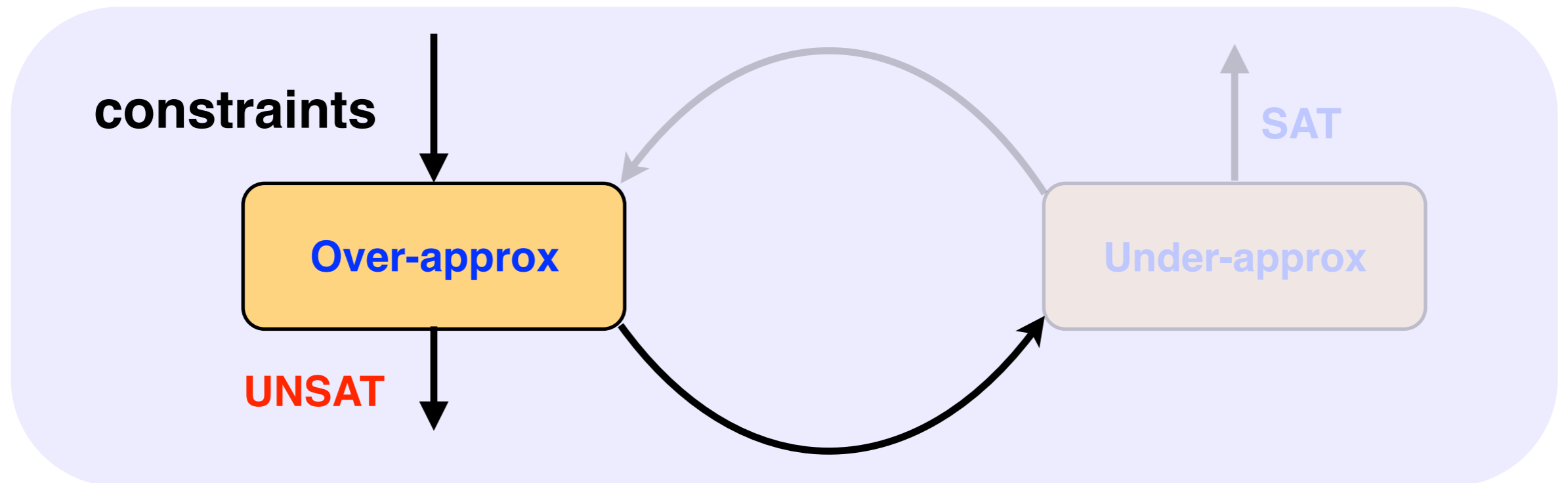
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# Overview



Using **CEGAR** for string constraint solving

# Overview



Using **CEGAR** for string constraint solving

# Running Example

Grammar

$S : a S b \mid S b \mid \epsilon$

$X, Y \in L(S)$

$X = "a" . Y$

$X = Z$

Membership

Equality

$S: aSb \mid Sb \mid \epsilon$   
 $X, Y \in L(S)$   
 $X = "a" . Y$   
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**Over-approximation**

counter-example

3 steps

Step 1: Over approximate CFG constraints to **regular constraints**

$S: aSb \mid Sb \mid \epsilon$   
 $X, Y \in L(S)$

$X, Y \in L(a^* b^*)$

Step 2: Rename each occurrence of variables in equalities

$X = "a" . Y$   
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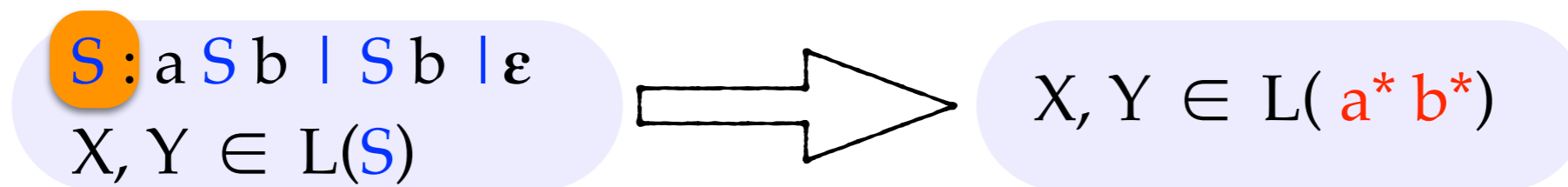
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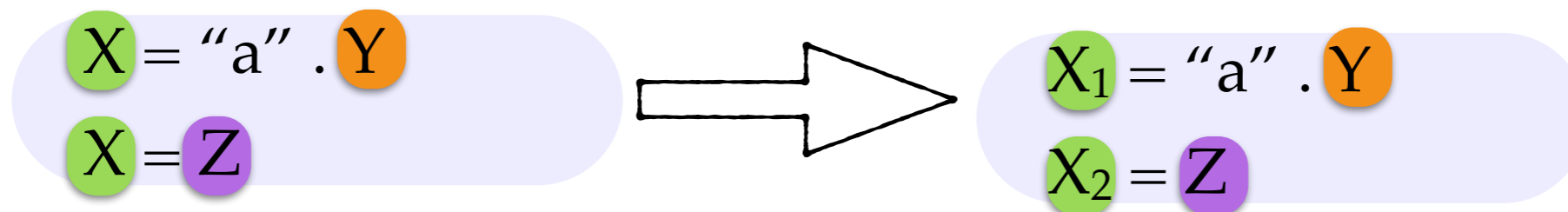
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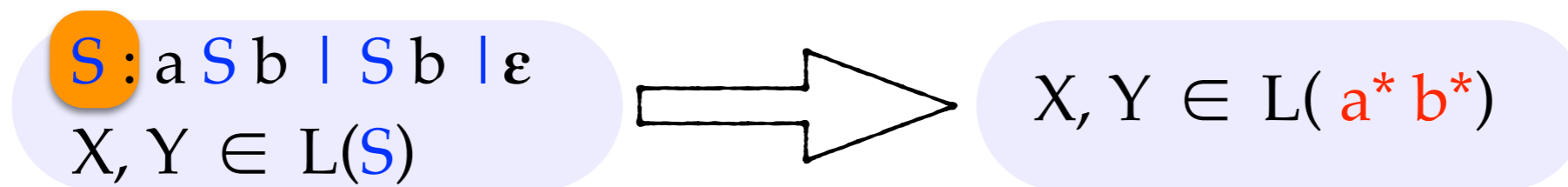


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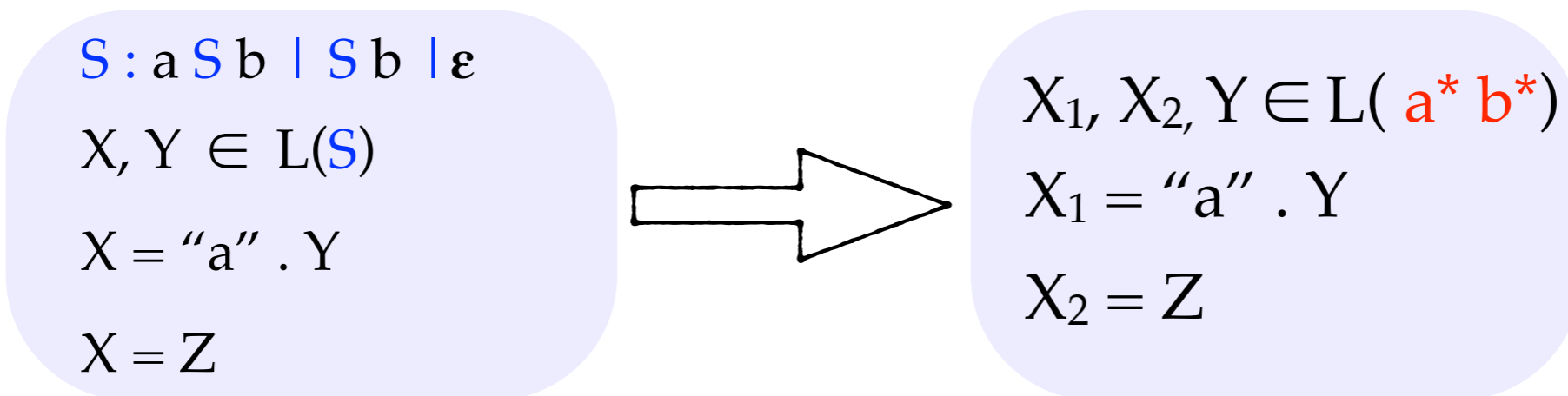
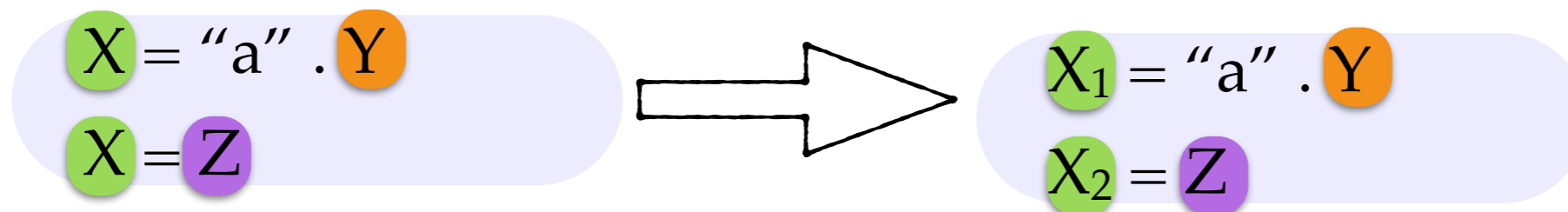
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The new constraints fall into decidable fragments, can be handled efficiently.

$X_1, X_2, Y \in L(a^* b^*)$   
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Norn solver  
[CAV14]

SAT

counter-example

correct

spurious

UNSAT

- original constraints **unsat**
- **terminate**

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**SAT**

counter-example  $\left[ \begin{array}{l} \text{correct} \\ \text{spurious} \end{array} \right.$

$X_1 = aa$     $X_2 = a$     $Y = a$     $Z = a$

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**Over-approximation**

$X_1 = aa \quad X_2 = a \quad Y = a \quad Z = a$

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[CAV14]

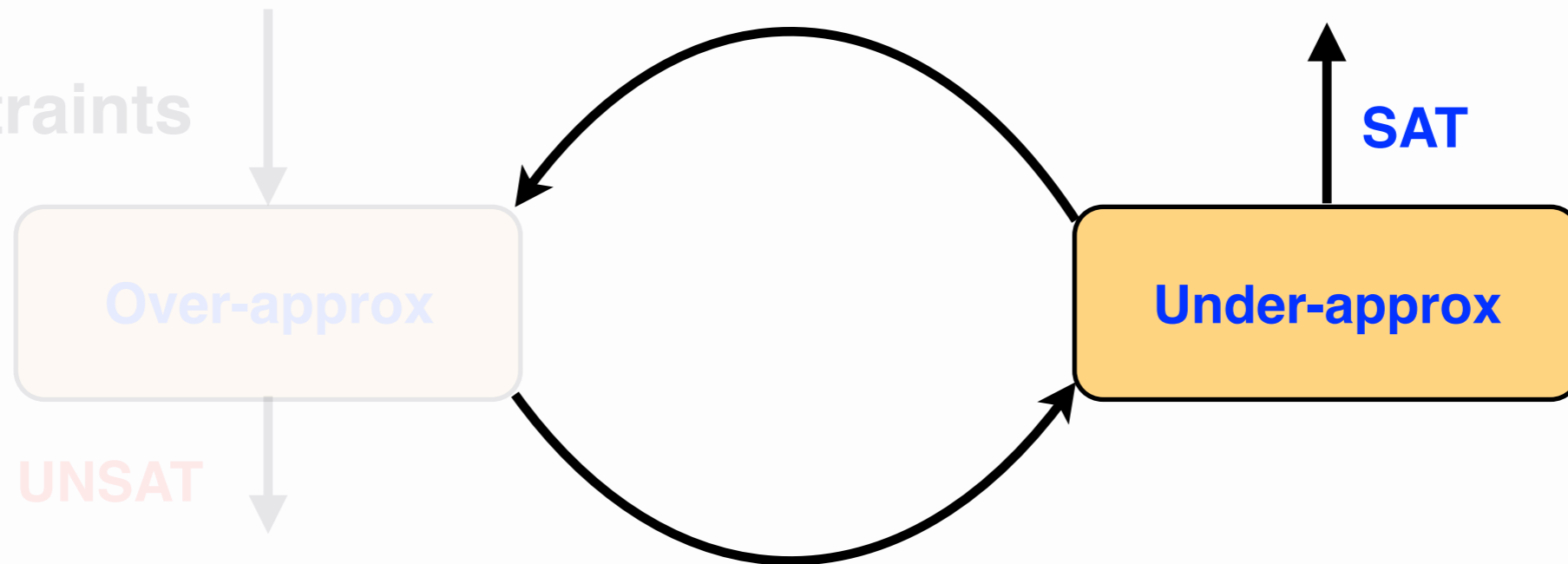
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# Overview

constraints



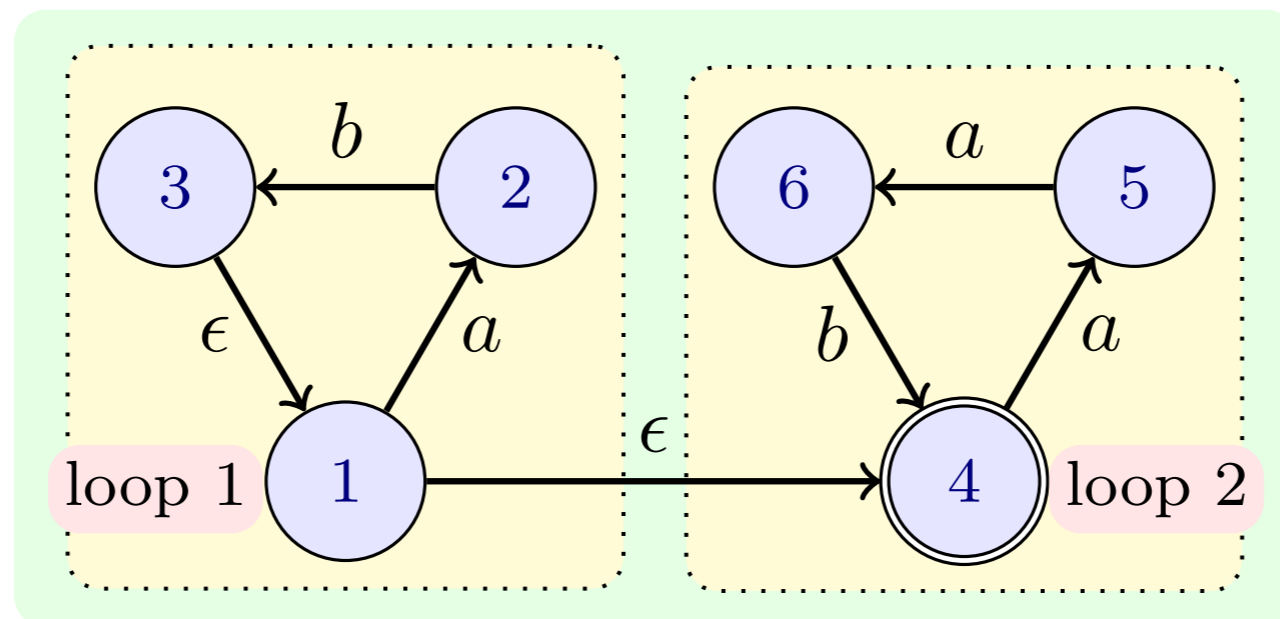
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# Flat Automata

basic concepts

## Definition:

- finite state automata
- consist of a sequence of simple loops



$(ab)^* (aab)^*$



$X_1 = aa$     $X_2 = a$     $Y = a$     $Z = a$

**Under-approximation**

**SAT** | **UNSAT**

4 steps

**Idea:** search for solutions accepted by **flat automata**

Step 1: Generate the minimal flat automaton that accepts the counter-example

Step 2: Intersect the constraints with the generated flat automaton

$S : a S b \mid S b \mid \epsilon$

$X, Y \in L(S)$

$X = "a" \cdot Y$

$X = Z$

$\cap$   $a^*$   $\rightarrow$

$X, Y \in L(\epsilon)$

$X = "a" \cdot Y$

$X = Z$

$X, Y, Z \in L(a^*)$

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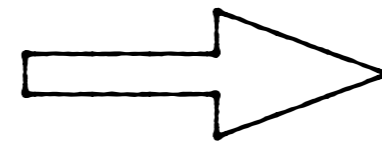
**SAT** | **UNSAT**

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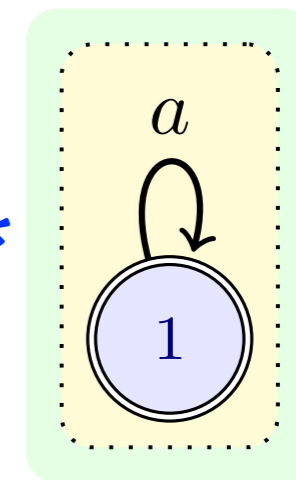
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$a^*$



Step 2: Intersect the constraints with the generated flat automaton

$S : aSb \mid Sb \mid \epsilon$

$X, Y \in L(S)$

$X = "a" \cdot Y$

$X = Z$



$a^*$



$X, Y \in L(\epsilon)$

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$Z \in L(a^*)$

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**Under-approximation**

**SAT** | **UNSAT**

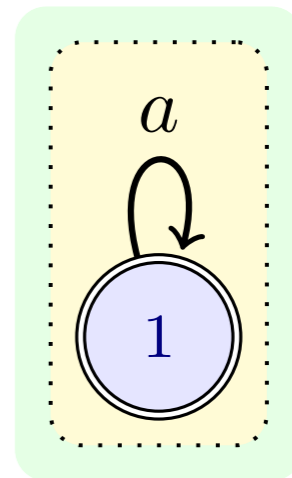
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$a^*$



Step 2: Intersect the constraints with the generated flat automaton  $a^*$

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$\cap a^*$

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$Z \in L(a^*)$

$X_1 = aa$     $X_2 = a$     $Y = a$     $Z = a$

**Under-approximation**

**SAT** | **UNSAT**

Step 3: Convert to quantifier-free Presburger formulas

$X, Y \in L(\epsilon)$

$X = "a" \cdot Y$

$X = Z$

$Z \in L(a^*)$

$|X| = 0$

$|Y| = 0$

$|X| = 1 + |Y|$

$|X| = |Z|$

$|X|, |Y|, |Z| \geq 0$

Step 4: Feed the formulas to a SMT solver

$X_1 = aa$     $X_2 = a$     $Y = a$     $Z = a$

**Under-approximation**

**SAT** | **UNSAT**

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**SMT**

**UNSAT**

# Overview

$a^*$  does not work

constraints

Over-approx

Under-approx

UNSAT

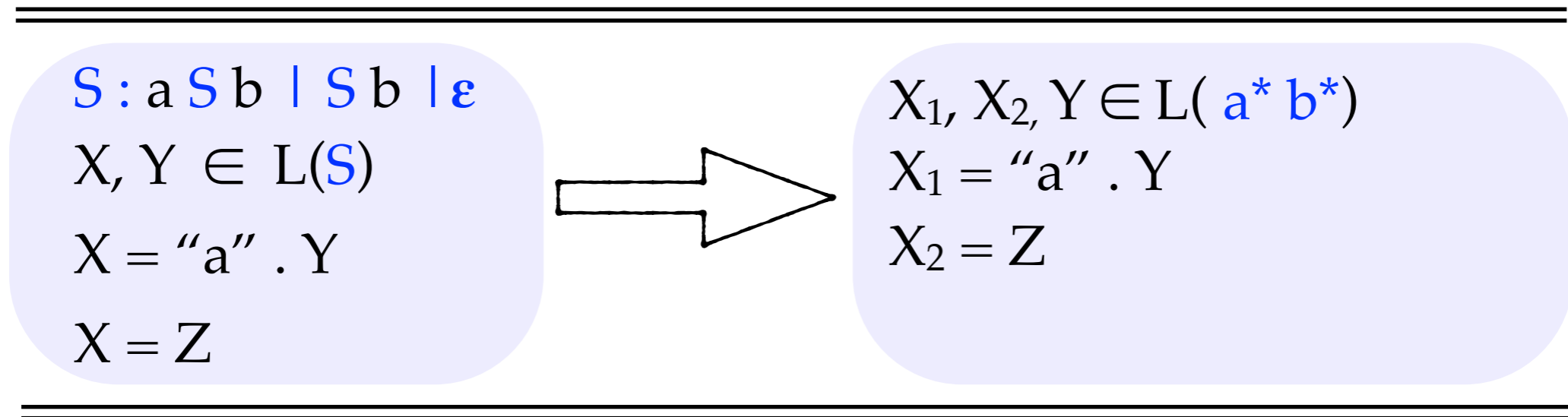
SAT

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Step 1: Over approximate CFG constraints to **regular constraints**

Step 2: Rename each occurrence of variables in equalities

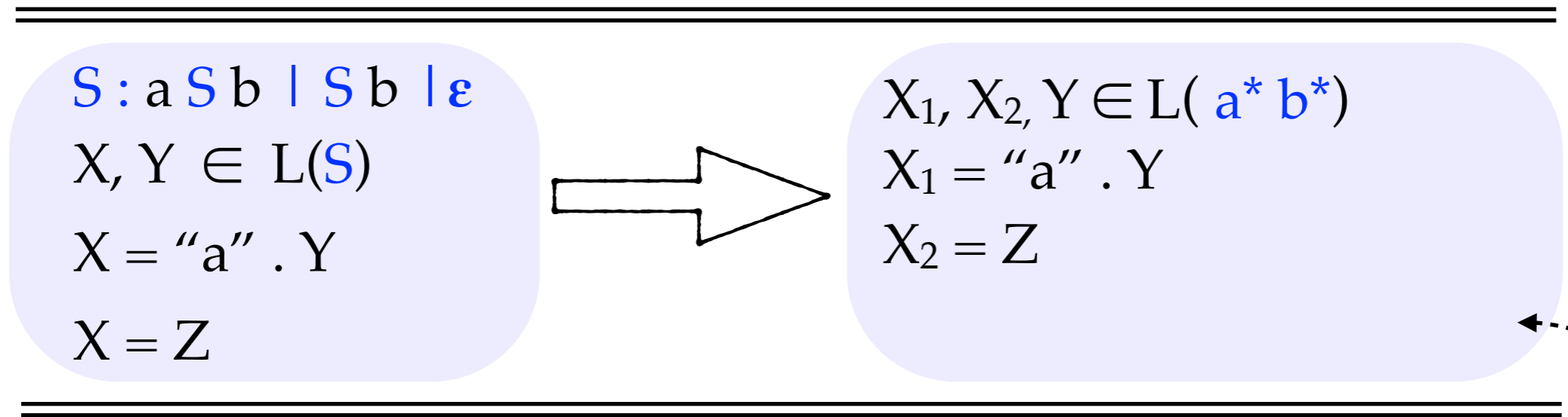


Step 3: Refine the over-approximation



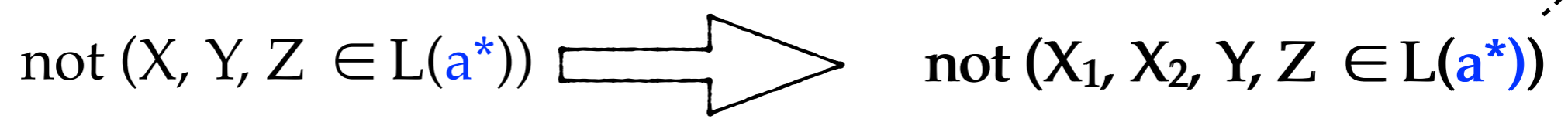
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Step 3: **Refine** the over-approximation

NEW





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 $X, Y \in L(S)$   
 $X = "a" \cdot Y$   
 $X = Z$

**Over-approximation**

counter-example

Step 4: Solve the approximate constraints

$X_1, X_2, Y \in L(a^* b^*)$   
 $X_1 = "a" \cdot Y$   
 $X_2 = Z$   
 $\text{not } (X_1, X_2, Y, Z \in a^*)$

Norn

**SAT**

**UNSAT**

$S: aSb \mid Sb \mid \epsilon$   
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Norn

**SAT**

counter-example [ correct  
spurious

$X_1 = aab \quad X_2 = aab \quad Y = ab \quad Z = aab$

$S: aSb \mid Sb \mid \epsilon$   
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**Over-approximation**

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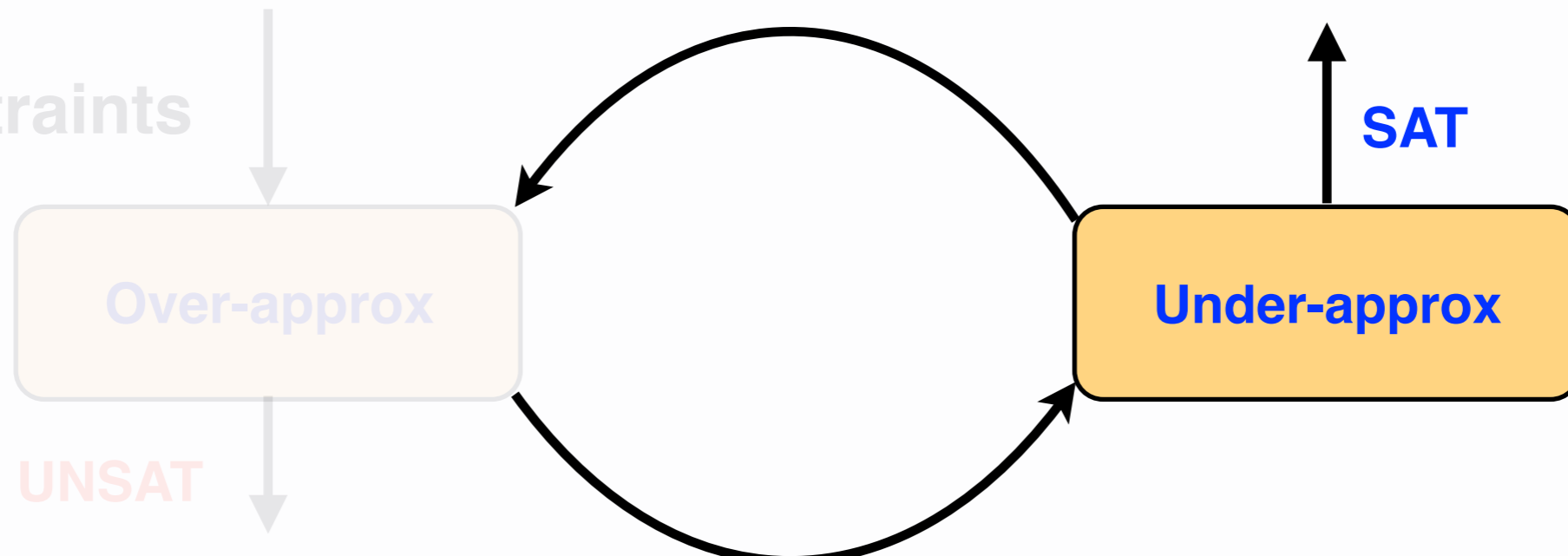
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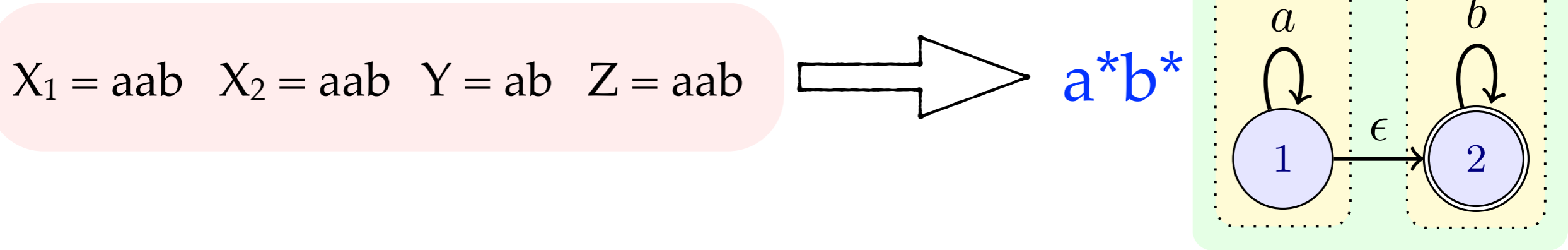
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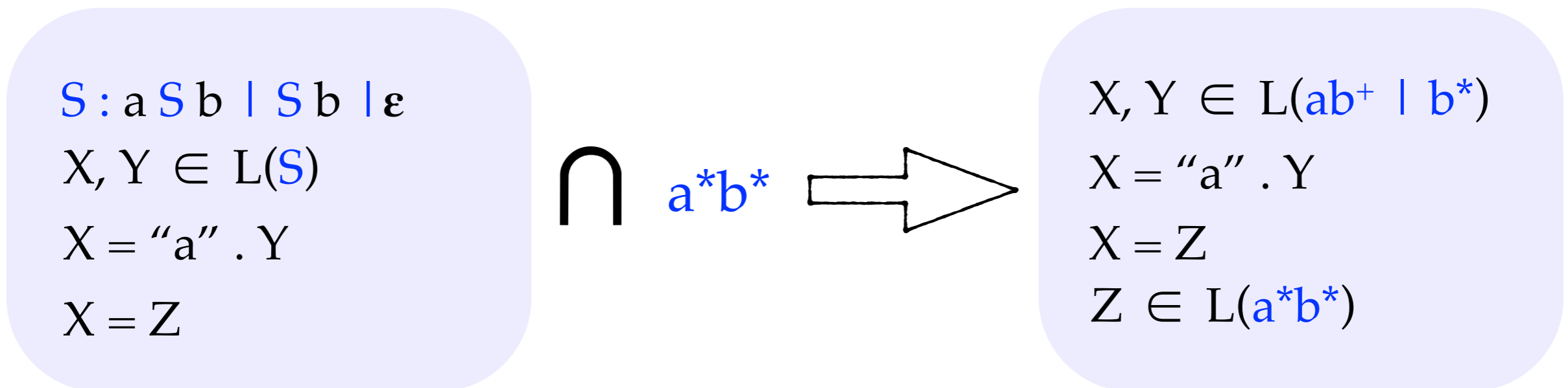
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Step 1: Generate the minimal flat automaton that accepts the counter-example



Step 2: Intersect the constraints with the generated flat automaton



$X_1 = aab$   $X_2 = aab$   $Y = ab$   $Z = aab$

2nd  
iteration

**Under-approximation**

**SAT** | **UNSAT**

Step 3: Convert to quantifier-free Presburger formulas

$X, Y \in L(ab^+ \mid b^*)$

$X = "a" \cdot Y$

$X = Z$

$Z \in L(a^*b^*)$

$X, Y \in L(ab^+ \mid b^*)$



$X \in L(ab^+)$  and  $Y \in L(ab^+)$

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2nd  
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Under-approximation

SAT | UNSAT

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$X \in L(ab^+)$  and  $Y \in L(ab^+)$

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$X_1 = aab$   $X_2 = aab$   $Y = ab$   $Z = aab$

2nd iteration

**Under-approximation**

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$Z \in L(a^*b^*)$



$X \in L(ab^+)$  and  $Y \in L(b^*)$

$X = "a" \cdot Y$

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$Z \in L(a^*b^*)$

$X, Y \in L(ab^+ \mid b^*)$



$X \in L(ab^+)$  and  $Y \in L(ab^+)$

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2nd iteration

Under-approximation

SAT | UNSAT

### Step 3: Convert to quantifier-free Presburger formulas

$X \in L(ab^+)$  and  $Y \in L(b^*)$

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$X = Z$

$Z \in L(a^*b^*)$



$|X| = 1 + |Y|$

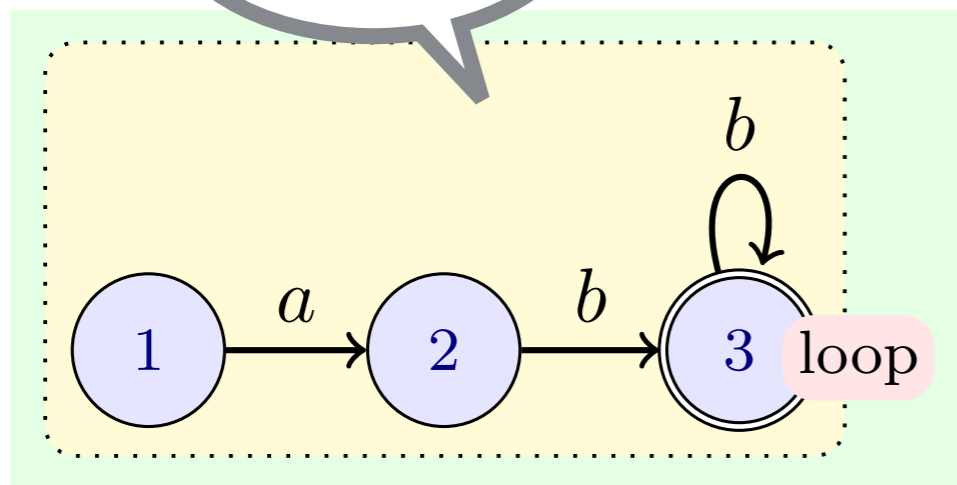
$|X| = |Z|$

$|X|, |Y|, |Z| \geq 0$

$\#Y("a") = 0, \#X("a") = 1$

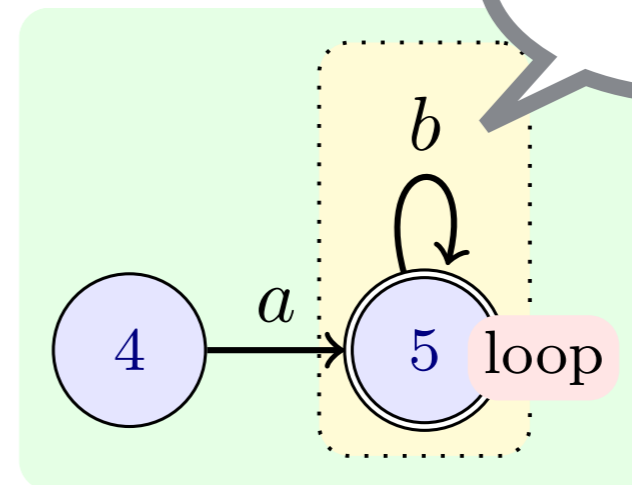
$\#Y("b") = \#X("b")$

$X \in L(ab^+)$



=

$Y \in L(b^*)$



$\#X("a")$ : number of occurrences of "a" in X

$X_1 = aab$   $X_2 = aab$   $Y = ab$   $Z = aab$

2nd iteration

Under-approximation

SAT | UNSAT

### Step 3: Convert to quantifier-free Presburger formulas

$X \in L(ab^+)$  and  $Y \in L(b^*)$

$X = "a" \cdot Y$

$X = Z$

$Z \in L(a^*b^*)$



$|X| = 1 + |Y|$

$|X| = |Z|$

$|X|, |Y|, |Z| \geq 0$

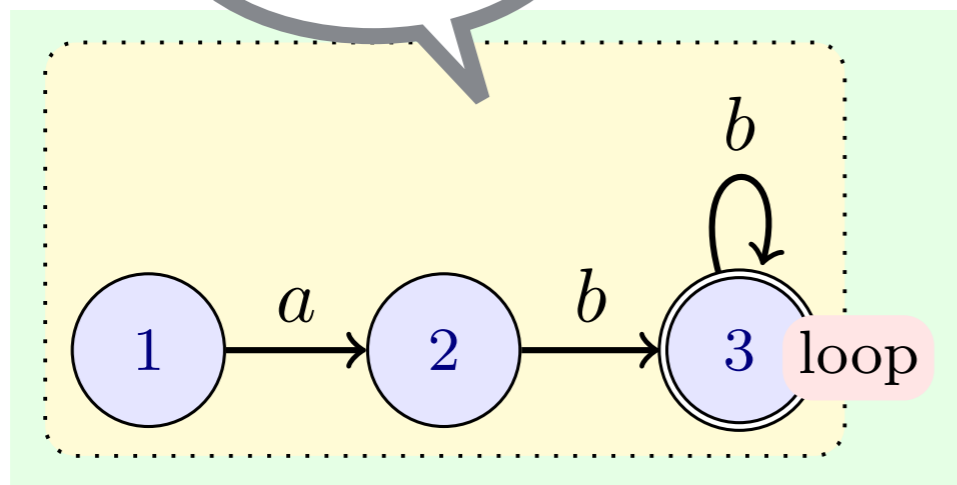
$\#Y("a") = 0, \#X("a") = 1$

$\#Y("b") = \#X("b")$

$\#Z("a") = \#X("a")$

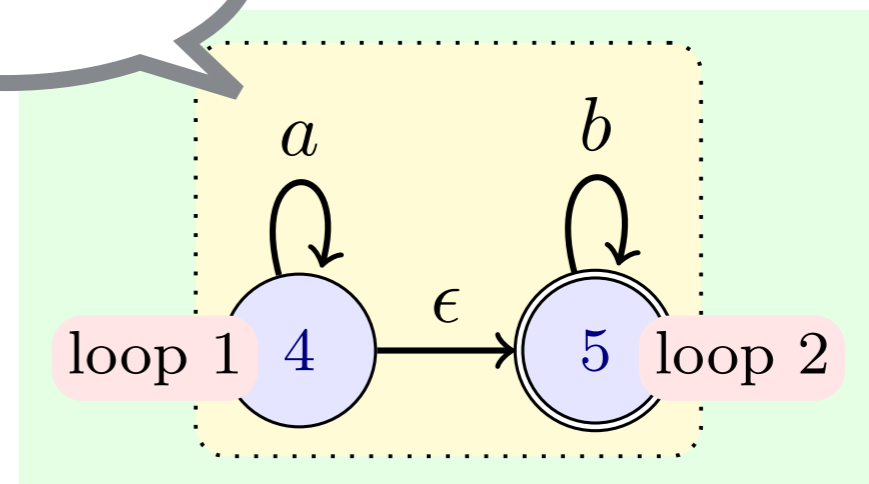
$\#Z("b") = \#X("b")$

$X \in L(ab^+)$



=

$Z \in L(a^*b^*)$



$\#X("a")$ : number of occurrences of "a" in X

$X_1 = aab$   $X_2 = aab$   $Y = ab$   $Z = aab$

2nd iteration

**Under-approximation**

**SAT** | **UNSAT**

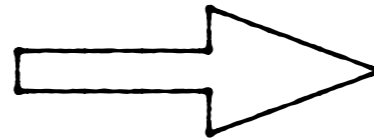
Step 3: Convert to quantifier-free Presburger formulas

$X \in L(ab^+)$  and  $Y \in L(b^*)$

$X = "a" \cdot Y$

$X = Z$

$Z \in L(a^*b^*)$



$|X| = 1 + |Y|$

$|X| = |Z|$

$|X|, |Y|, |Z| \geq 0$

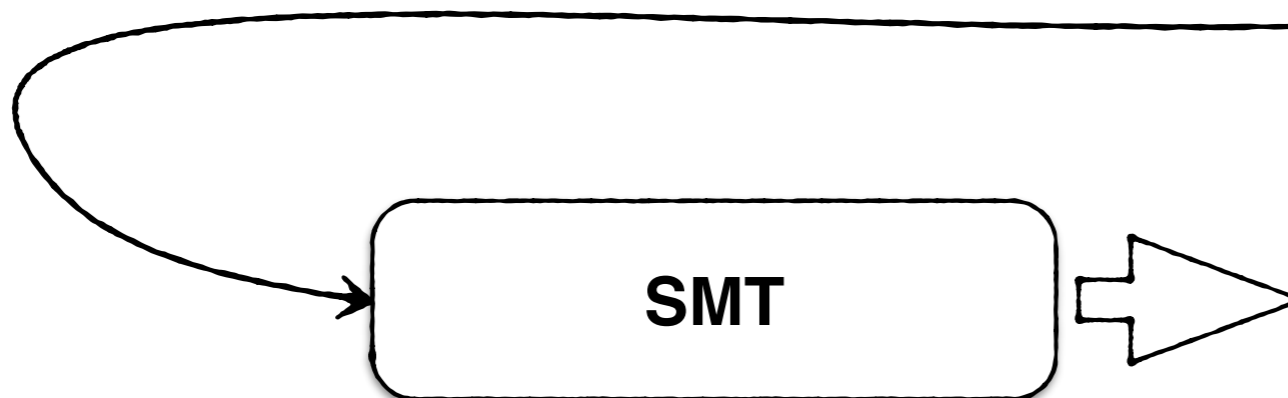
$\#Y("a") = 0, \#X("a") = 1$

$\#Y("b") = \#X("b")$

$\#Z("a") = \#X("a")$

$\#Z("b") = \#X("b")$

Step 4: Feed the formulas to a SMT solver



**SAT**

$\#X("b") = 1$

$\#Y("b") = 1$

$\#Z("a") = 1$

$\#Z("b") = 1$

$X = ab$

$Y = b$

$Z = ab$

# Experiment Results



- ✓ Open-source tool: TRAU
- ✓ Use Z3 as a backend tool
- ✓ Run on the standard **Kaluza** & **SQL injection** benchmarks
  - **Kaluza**: ~50,000 tests  
Javascript symbolic execution engine
  - **SQL injection**: 10 tests  
detect SQL injections with CFG constraints

# Experiment Results

## Kaluza benchmark result

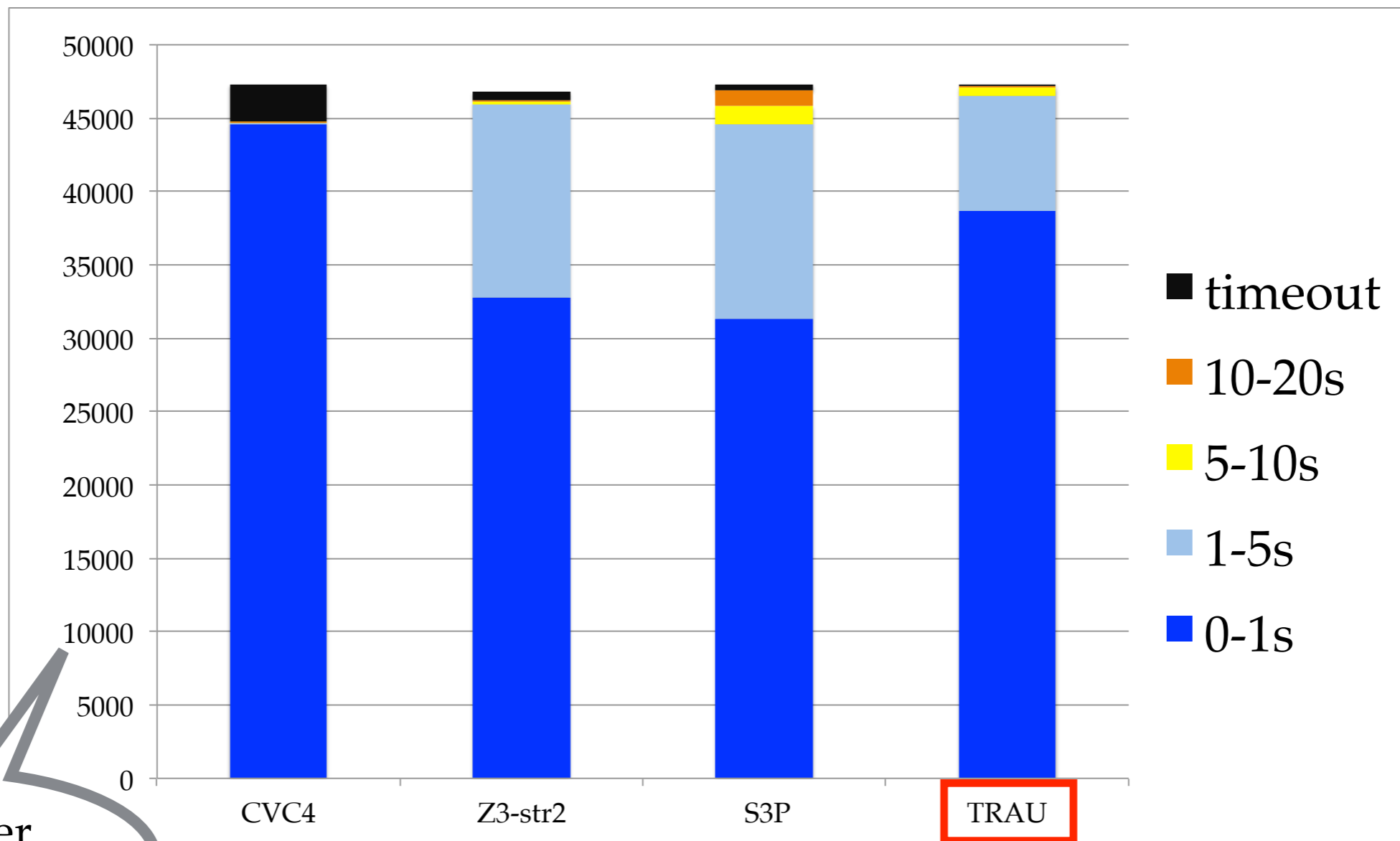
	CVC4	Z3-str2	S3P	TRAU
sat	33191	34459	34829	35202
unsat	11625	11747	12033	12019
timeout	2468	553	422	63

timeout  
20s

No.  
finished tests

# Experiment Results

## Kaluza benchmark result



Number  
of tests

# Experiment Results

## SQL Injection result

length bound for vars

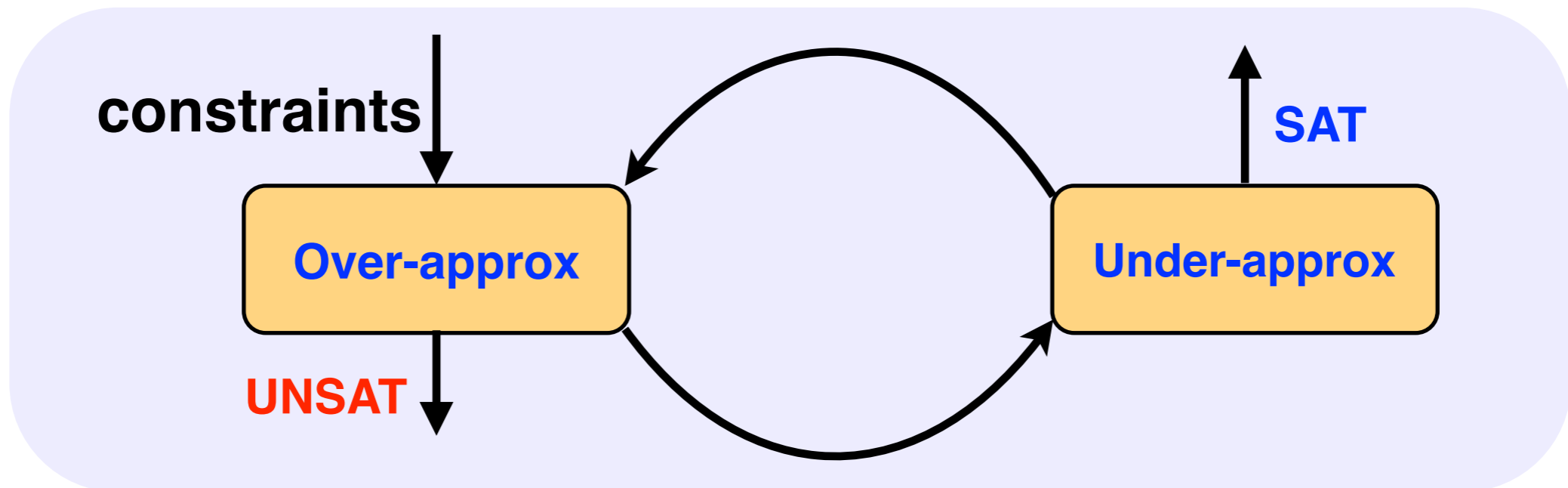
Input	Var	Length	TRAU				HAMPI	
			Bounded Length Result	Time(s)	Unbounded Length Result	Times(s)	Bounded Length Result	Times(s)
cfg01	6	20	sat	1.14	sat	1.24	sat	0.52
cfg02	6	20	unsat	1.02	unsat	1.11	unsat	0.20
cfg03	8	50	sat	1.01	sat	1.45	sat	9.34
cfg04	8	50	unsat	1.56	unsat	1.54	unsat	9.33
cfg05	10	70	sat	1.55	sat	2.00	-	timeout
cfg06	10	70	unsat	2.01	unsat	1.12	-	timeout
cfg07	14	50	sat	2.13	sat	3.36	-	timeout
cfg08	14	50	unsat	1.56	unsat	2.58	unsat	8.85
cfg09	20	70	sat	1.78	sat	2.27	-	timeout
cfg10	20	70	unsat	2.46	unsat	1.89	-	timeout

20s

# Summary

## 1. New framework for solving **string constraints**:

- Handle **rich** class of constraints: CFG membership, transducer, etc.
- Based on Counter-Example Guided Abstract Refinement.



## 2. Open-source tool: **outperforming** all existing tools.





**Thank you!**